Exam 04/02/2020

## Problem

Please open the file Pisa_00_03_06_09 _12_15_18.sav.
a) Please run a $k$-means cluster analysis with the three variables "Lesekompetenz (Reading literacy)", "'Mathematische Grundbildung (Mathematical literacy)", ""Naturwissenschaftliche Grundbildung (Scientific literacy)", for the competition in the year 2018. As the number of clusters please select the value four.

1. How many cases are in each cluster? Please complete the following table:

\left.|  | Cluster |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | :---: |
|  |  | 1 | 2 | 3 |  |$\right) 4$

2. Please complete the following table:
average value

|  | Cluster |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
|  | 1 | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| Reading Literacy |  |  |  |  |
| Mathematical Literacy |  |  |  |  |
| Scientific Literacy |  |  |  |  |

3. Please comment the four clusters.
4. Which cluster does Germany belong to?
5. Which cluster does PSJZ China belong to?
b) Run a hierarchical cluster analysis with the three variables "Lesekompetenz (Reading literacy)"', "'Mathematische Grundbildung (Mathematical literacy)"', "'Naturwissenschaftliche Grundbildung (Scientific literacy)"' for the competition in the year 2018.
6. How many clusters should be constructed? (Give reasons!)
7. Which cluster does Germany belong to?
8. Which cluster does PSJZ China belong to?
c) Please class the cases of the two variables " ${ }^{\text {M Mathematische Grundbildung }}$ (Mathematical literacy)", and "'Naturwissenschaftliche Grundbildung (Scientific literacy)", for the competition in the year 2018 into three classes. The classes should have about the same number of cases. Please check with a level $\alpha=0.05$ test, whether the two classed variables are stochastically independent.
9. What is the name of the test?

## 2. Is the rule of thumb fulfilled? (Give reasons!)

3. How small is the $p$-value?
4. What is the test decision? (Comment!)
5. Compute a measure of association between the classed variables.Comment!
d) What is a measure to verify the loss of information if the data set is plotted in scatterplot, where $x$-axis and $y$-axis are the first and the second principal components of the three variables "'Lesekompetenz (Reading literacy)",' "'Mathematische Grundbildung (Mathematical literacy)", "'Naturwissenschaftliche Grundbildung (Scientific literacy)" ' for the competition in the year 2018? Compute and comment the value of this measure.

Problem 31.01.2019
Please open the file Credit_card.sav.
a) Please run a hierarchical cluster analysis with the two variables "items" and "spent".

1. Please complete the following table:

| Stage | Coefficients |
| :---: | :--- |
| 26277 |  |
| 26278 |  |
| 26279 |  |

2. How many clusters should be constructed? (Explain!)
b) Please run a K-Means cluster analysis with the two variables "'items", and "'spent"'. Three clusters should be constructed.
3. Please complete the following table:

| average value |  |  |  |
| :--- | ---: | ---: | ---: |
|  | Cluster |  |  |
|  |  | 1 | 2 |
| Number of items |  |  | 3 |
| Amount spent |  |  |  |
| Number of cases |  |  |  |

2. Please comment the three clusters.
c) Please run a level 0.05 test to check whether gender and clustermembership are dependent. Proceed as follows:
3. What is the name of the test?
4. Please check the rule of thumb of the test.
5. Please give the value of the $p$-value?
6. What is the test decision? (Comment!)
7. Please compute the measure of association Gamma between gender and clustermembership. What is the value of Gamma?

## Technology Arts Sciences Cologne

Faculty of Economics, Business and Law
Prof. Dr. Arrenberg
Room 221, Tel. 3914
jutta.arrenberg@th-koeln.de

## Master: Quantitative Methods

Old Exams
Problem (25.01.2017)
Please open the file telco_extra.sav.
a) Please run a hierarchical cluster analysis with the five variables Month with service, Age in years, Years at current address, Household income in thousands, Number of people in household. How many clusters should be constructed? (Give reasons!)
b) Please run a $k$-means cluster analysis with the five variables Month with service, Age in years, Years at current address, Household income in thousands, Number of people in household. As the number of clusters please select the value three.

1. How many cases are in cluster 1 resp. 2 resp. 3?

|  | Cluster |  |  |
| :--- | ---: | ---: | ---: |
|  | 1 | 2 | 3 |
|  |  |  |  |

2. Please complete the following table:

Average Values

|  | Cluster |  |  |
| :--- | :--- | ---: | ---: |
|  | 1 |  | 2 |
| Month with service |  |  |  |
| Age in years |  |  |  |
| Years at current address |  |  |  |
| Household income in thousands US-Dollar |  |  |  |
| Number of people in household |  |  |  |

3. Please comment the three clusters.
4. Check with a statistical test whether the medians of the variable "Years with current employer" are the same across the three clusters. What is the name of the test? What are the assumptions of the test? What is the $p$-value of the test? What indicates this $p$-value?
5. What are the three values of the empirical medians of the variable "Years with current employer" in the clusters?

Empirical Medians

|  | Cluster |  |  |
| :---: | ---: | ---: | ---: |
|  | 1 | 2 | 3 |
| Years with <br> current employer |  |  |  |

c) Suppose you will run an ordinal regression for the dependent variable $Y=$ "Calling card last month". What is the link function if the variable $Y$ is transformed into an ordinal leveled variable with the three categories " 0 up to 10 ", "more than 10 up to 20 " and "more than 20 "? (Give reasons!)

Exam QM 04/02/2020

Pisa Survey 2018

## Number of Cases in each

## Cluster

| Cluster | 1 | 32,000 |
| :--- | ---: | ---: |
|  | 2 | 9,000 |
|  | 3 | 25,000 |
| Valid |  | 11,000 |
| Missing |  | 77,000 |

## Final Cluster Centers

|  | Cluster |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
|  | 1 | 2 | 3 | 4 |  |
| Reading | 485,06 | 363,78 | 414,40 | 522,64 |  |
| Mathematical | 488,72 | 367,56 | 417,36 | 537,36 |  |
| Scientific | 485,56 | 372,11 | 420,08 | 531,55 |  |

Cluster 1: second best performance
Cluster 2: poorest performance
Cluster 3: third best performance
Cluster 4: best performance

Germany belongs to cluster 1 .
China belongs to cluster 4.

| Stage | Coefficients |
| :--- | :--- |
| 75 | 35.057 |
| 76 | 45.177 |

The greatest jump of the coefficients happens from stage 75 to stage 76 .
Recommended number of clusters $=\mathrm{N}-75=77-75=2$.
Germany belongs to cluster 1 .
China belongs to cluster 2.

Count

|  |  | Percentile Group of Naturw_2018 |  |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 |  |
| Percentile Group of | 1 | 22 | 3 | 0 | 25 |
| Mathe_2018 | 2 | 4 | 19 | 4 | 27 |
|  | 3 | 0 | 4 | 22 | 26 |
| Total |  | 26 | 26 | 26 | 78 |

## Chi-Square Tests

|  | Value | df | Asymptotic Significance (2sided) |
| :---: | :---: | :---: | :---: |
| Pearson Chi-Square | 82,519 ${ }^{\text {a }}$ | 4 | ,000 |
| Likelihood Ratio | 86,807 | 4 | ,000 |
| Linear-by-Linear Association | 56,225 | 1 | ,000 |
| N of Valid Cases | 78 |  |  |

a. 0 cells $(0,0 \%)$ have expected count less than 5 . The minimum expected count is 8,33 .

|  |  | Symmetric Measures |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Value | Asymptotic Standard Error ${ }^{\text {a }}$ | Approximate $T^{\text {b }}$ | Approximate Significance |
| Ordinal by Ordinal | Gamma | ,967 | ,017 | 18,509 | ,000 |
| $N$ of Valid Cases |  | 78 |  |  |  |

a. Not assuming the null hypothesis.
b. Using the asymptotic standard error assuming the null hypothesis.

Classed Points in science and mathematics are dependent.

Gamma=0.967 positive strong relationship between mathematical literacy and scientific literacy. High points in mathematics are going along with high points in science.

## Total Variance Explained

| Component | Total | Initial Eigenvalues |  | Extraction Sums of Squared Loadings |  |  | Rotation Sums of Squared Loadings |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | \% of <br> Variance | Cumulative \% | Total | \% of Variance | Cumulative \% | Total | \% of <br> Variance | Cumulative $\%$ |
| 1 | 2,932 | 97,740 | 97,740 | 2,932 | 97,740 | 97,740 | 1,526 | 50,865 | 50,865 |
| 2 | ,052 | 1,745 | 99,485 | ,052 | 1,745 | 99,485 | 1,459 | 48,620 | 99,485 |
| 3 | ,015 | ,515 | 100,000 |  |  |  |  |  |  |

Extraction Method: Principal Component Analysis.
99.485 \% of the total variance is explained by the first two principal components.

## Exam 31/01/2019

Credit_card.sav

| Stage | Coefficient |
| :--- | :--- |
| 26277 | 31.226 |
| 26278 | 75.160 |
| 26279 | 117.824 |

$75.160-31.226=43.934$
$117.824-75.160=42.664$

Number of Clusters $=\mathrm{n}-26277=26280-26277=3$ Cluster

Final Cluster Centers

|  | Cluster |  |  |
| :--- | ---: | ---: | ---: |
|  | 1 | 2 | 3 |
| Number of items | 4 | 7 | 1 |
| Amount spent | 294,38 | 625,90 | 48,92 |

Cluster 1 = medium number of items, medium amount spent
Cluster 2 = highest number of items, highest amount spent
Cluster 3 = smallest number of items, smallest amount spent

## Cluster Number of Case * Gender Crosstabulation

Count

|  | Gender |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
|  |  | Male | Female | Total |
| Cluster Number of Case | 1 | 4330 | 4089 | 8419 |
|  | 2 | 1582 | 1547 | 3129 |
|  | 3 | 7528 | 7204 | 14732 |
| Total |  | 13440 | 12840 | 26280 |

Chi-Square Tests
$\left.\begin{array}{lr|r|rr} & & & \begin{array}{c}\text { Asymptotic }\end{array} \\ \text { Significance (2- } \\ \text { sided) }\end{array}\right]$
a. 0 cells $(0,0 \%)$ have expected count less than 5 . The minimum expected count is 1528,78 .
$p$-value $=0.698>0.05$
No rejection of the null hypothesis, i.e. Gender and Cluster Membership are stochastically independent.

## QCL_ordered * Gender Crosstabulation

Count

|  | Gender |  |  |  |
| :--- | :--- | ---: | ---: | ---: |
|  | Male | Female | Total |  |
| QCL_ordered | low number items, low <br> amount spent | 7528 | 7204 | 14732 |
| medium no items, medium <br> amount spent | 4330 | 4089 | 8419 |  |
| high no items, high amount <br> spent | 1582 | 1547 | 3129 |  |
| Total |  | 13440 | 12840 | 26280 |

X= Gender (male, female) dichotomous variable
$\mathrm{Y}=$ Cluster membership (1=medium group, 2=top group, $3=$ poor group) nominal
Recoding Cluster membership into an ordinal leveled variable 1=poor group, 2=medium group, $3=$ top group) for to calculate gamma. Gamma $=0.001$

|  |  | Symmetric Measures |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Value | Asymptotic Standard Error ${ }^{\text {a }}$ | Approximate $\mathrm{T}^{\text {b }}$ | Approximate Significance |
| Ordinal by Ordinal | Gamma | ,001 | ,011 | ,062 | ,951 |
| $N$ of Valid Cases |  | 26280 |  |  |  |

a. Not assuming the null hypothesis.
b. Using the asymptotic standard error assuming the null hypothesis.
a) Hierarchical cluster analysis

Greatest jump oft he coefficients

| Stage | Coefficient |
| :--- | :--- |
| 998 | 197.601 |
| 999 | 537.970 |

Number of cluster $=\mathrm{n}-998=1000-998=2$ cluster
b) K-means cluster analysis
1.

## Number of Cases in each

## Cluster

| Cluster | 1 | 894,000 |
| :--- | :--- | ---: |
|  | 2 | 6,000 |
|  | 3 | 100,000 |
| Valid |  | 1000,000 |
| Missing |  | , 000 |

2. 

## Final Cluster Centers

|  | Cluster |  |  |  |
| :--- | ---: | ---: | ---: | ---: |
|  | 1 | 2 |  | 3 |
| Months with service | 34 | 59 | 49 |  |
| Age in years | 40 | 60 | 53 |  |
| Years at current address | 11 | 29 | 17 |  |
| Household income in <br> thousands | 51,70 | 1012,83 | 252,37 |  |
| Number of people in <br> household | 2 |  | 1 | 2 |

3. Cluster 1: youngest average age, shortest time of service and at current address, lowest income, average 2 people in household Cluster 2: oldest average age, longest time of service and at current address, highest income, single household
Cluster 3: median average age, median time of service and at current address, median income, average 2 people in household
4. Kruskal-Wallis test

Stochastic independence of „Years with current employer" across the three cluster
$p$-value $=0.000$
Rejection of H0, at least two medians of „Years with current employer" differ significantly across the three cluster
5.

## Report

## Median

Years with

| Cluster Number of Case | current employer |
| :--- | ---: |
| 1 | 7,00 |
| 2 | 32,00 |
| 3 | 26,00 |
| Total | 8,00 |

c)

| Class | Cases |
| :---: | :--- |
| $\leq 10$ | 434 |
| $10-\leq 20$ | $308=742-434$ |
| $>20$ | 258 |

Link function: negative log log

